Coordinate Geometry Proof Practice

Tips for doing Coordinate Geometry Proofs:

- **Organize** your work and **label everything**. Do not just perform calculations all over the place and leave your teacher to figuring out what is what (because we won't!).
- label your algebra statements clearly
  - so, for example, if you're going to prove the figure on the next page is a parallelogram by definition, one thing you'll need to do is find the slope of $\overline{BC}$. When you show that, write something like $slope\overline{BC} = \frac{3-0}{-4-8} = \frac{3}{-12} = \frac{-1}{4}$.
- you must refer to your calculations and provide a **summary/proof statement** when done. So, for example, if you have just finished finding 4 slopes and are now ready to say that it is a parallelogram, then you would finish with something like this:
  - $\overline{BC} \parallel \overline{AD}$ because both have slopes = -1/4
  - $\overline{AB} \parallel \overline{CD}$ because both have slopes = 4/1
  - since both pairs of opp. sides are ||, it's a ▱ by def. ✓
- do **NOT** turn nice fractions like ¾ into decimals – reduce all fractions
- you must **show algebraic work** for things in your proofs – you can not just simply, for example, look at the graph paper and write down the pt. where it looks like 2 lines intersect – you must use some algebraic way to find the point

Here is some warm-up/review for the proofs on the following pages:

1) What is the equation of the line that goes through (1, 3) and (5, 12)? Leave your answer in point-slope form.

   slope $= \frac{12-3}{5-1} = \frac{9}{4}$ $\quad y-3 = \frac{9}{4}(x-1)$

   $y = \frac{9}{4}x - \frac{9}{4} + 3$

   $\boxed{y = \frac{9}{4}x + \frac{3}{4}}$ ✓

2) What is the midpoint of (1, 3) and (5, 12)?

   $\left(\frac{1+5}{2}, \frac{3+12}{2}\right) = \boxed{\left(3, \frac{15}{2}\right)}$ ✓

3) What is the distance between (1, 3) and (5, 12)?

   $d = \sqrt{(1-5)^2 + (3-12)^2} = \sqrt{(-4)^2 + (-9)^2} = \sqrt{16+81} = \boxed{\sqrt{97}}$ ✓

4) What is the equation of the line that is || to the line in #1 and also goes through (0, -1)?

   same slope of 9/4 $\quad y-(-1) = \frac{9}{4}(x-0)$ $\quad \boxed{y = \frac{9}{4}x - 1}$ ✓

5) What is the equation of the line that is ⊥ to the line in #1 and also goes through (0, -1)?

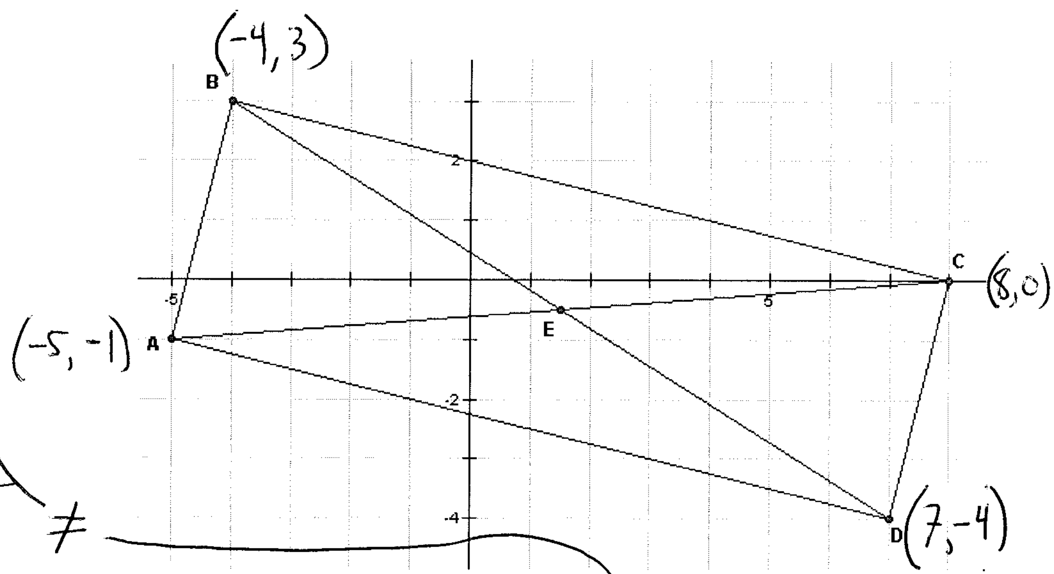
   neg. recip. of $\frac{9}{4}$ is $\frac{-4}{9}$ $\quad y-(-1) = \frac{-4}{9}(x-0)$

   $\boxed{y = \frac{-4}{9}x - 1}$ ✓

$$AB = \sqrt{(-4 - -5)^2 + (3 - -1)^2} = \sqrt{1 + 16} = \sqrt{17}$$

$$AD = \sqrt{(-5-7)^2 + (-1 - -4)^2} = \sqrt{(-12)^2 + 3^2} = \sqrt{144+9} = \sqrt{153}$$

$\neq$



1. Given the figure above, prove that it is specifically a **rectangle** and not a square. There are many ways to do this. Let's practice a few. Prove it's a rectangle by:

- showing it's a parallelogram with one right angle and 2 sides are not ≅.

so, it's a rectangle & can't be a square ✓

$$\text{slope } \overline{BC} = \frac{3-0}{-4-8} = \frac{3}{-12} = \frac{-1}{4}$$

so, $\overline{BC} \parallel \overline{AD}$

$$\text{slope } \overline{AD} = \frac{-4--1}{7--5} = \frac{-3}{12} = \frac{-1}{4}$$

so, by def., $\boxed{ABCD \text{ is a } \square}$

$$\text{slope } \overline{AB} = \frac{3--1}{-4--5} = \frac{4}{1}$$

so, $\overline{AB} \parallel \overline{CD}$

$$\text{slope } \overline{CD} = \frac{0--4}{8-7} = \frac{4}{1}$$

$\text{slope}\overline{AB} \cdot \text{slope } \overline{AD} = \frac{4}{1} \cdot \frac{-1}{4} = -1$ so, $\boxed{\overline{AB} \perp \overline{AD}}$

- showing that the diagonals are congruent and bisect each other and 2 sides are not ≅

$$AC = \sqrt{(-5-8)^2 + (-1-0)^2} = \sqrt{(-13)^2 + 1} = \sqrt{170}$$

$$BD = \sqrt{(-4-7)^2 + (3--4)^2} = \sqrt{(-11)^2 + 7^2} = \sqrt{170}$$

so, diags are ≅ (but that does not make it a $\square$)

$\boxed{\text{cont'd on last page *}}$

on the last page we showed the E is the m.p. of $\overline{AC}$ and $\overline{BD}$ - therefore, $\overline{AC}$ and $\overline{BD}$ bisect each other - we've also shown they are ≅ and that $\overline{AB} \not\cong \overline{AD}$ - so we're finally done. ✓ :)

- showing that the quadrilateral has 4 right angles and the diagonal are not ⊥

$\overline{AB} \perp \overline{AD}$ since slope product is -1 from above ✓

similarly, $\overline{AD} \perp \overline{CD}$, $\overline{CD} \perp \overline{BC}$, and $\overline{BC} \perp \overline{BA}$ since all products of slopes are -1. ✓

so, all ∠s are rt. by def.

slope $\overline{BD}$ = 7/-11

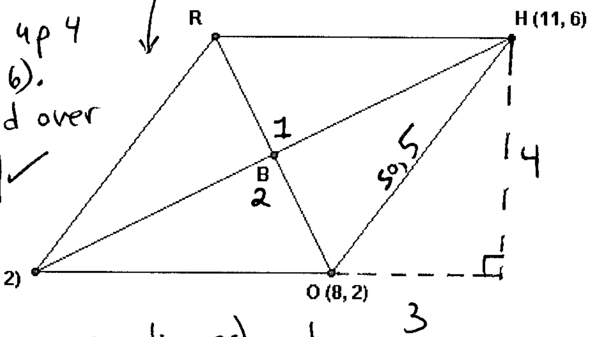
slope $\overline{AC}$ = 1/13

$\frac{7}{-11} \cdot \frac{1}{13} \neq -1$ so diags. are not ⊥

so it must be a rectangle

another nice rt. Δ on this side

2. If the above quadrilateral is a rhombus, what is the coordinate of R (show how you get it and say why your method works)?



Slope of $\overline{OH}$ is $\frac{2-6}{8-11} = \frac{4}{3}$ So, if I apply the slope once and go up 4 and over 3 from (8,2), I land at (11,6). By analogy, let me then go up 4 and over 3 from (3,2) to land on $\boxed{(6,6) = R.}$ ✓

a) now prove that RHOM is a rhombus

Since we already know how to use the dist. formula, let's use another part of our logical brain to solve this, We're dealing with nice lattice points (points with integer coordinates) and 2 sides lie horizontally - it's easy to see that MO = 5 and RH = 5. Now, to see that RM = HO = 5, look at the auxiliary lines I drew on the right side to form a rt. Δ. You can now see the beautiful Pathagorean Tripple (3,4,5). Since the other side has the same rt. Δ, all sides now equal 5, It's a rhombus ✓

b) find the coordinates of B (do so by finding equations for $\overleftrightarrow{RO}$ and $\overleftrightarrow{HM}$ and solving the system of 2 equations)

Slope $\overline{RO} = \frac{6-2}{6-8} = \frac{4}{-2} = -2$ It's a rhombus - so diags are ⊥. So, slope of $\overline{MH}$ is $\frac{1}{2}$.

$y-6 = -2(x-6)$
$y = -2x+18$

$y-2 = \frac{1}{2}(x-3)$
$y = \frac{1}{2}x + \frac{1}{2}$

$-2x+18 = \frac{1}{2}x + \frac{1}{2}$
$-4x+36 = x+1$
$35 = 5x$
$7 = x$ so, $y = -2(7)+18$
$= 4$

$\boxed{(7,4) = B}$ ✓

c) give another way to do problem b) and explain

Slope $\overline{MH} = \frac{1}{2}$ so, if you were to use the slope to get from M to pt. H, you'd have to apply the slope 4 times, To get from M to B then, apply it twice - go up 1 over 2 twice and you'll be at (7,4) ✓

d) prove that Δ RBH ≅ ΔOBM (again, using coordinate geometry)

slope of $\overline{RO} = -2$
slope of $\overline{MH} = \frac{1}{2}$

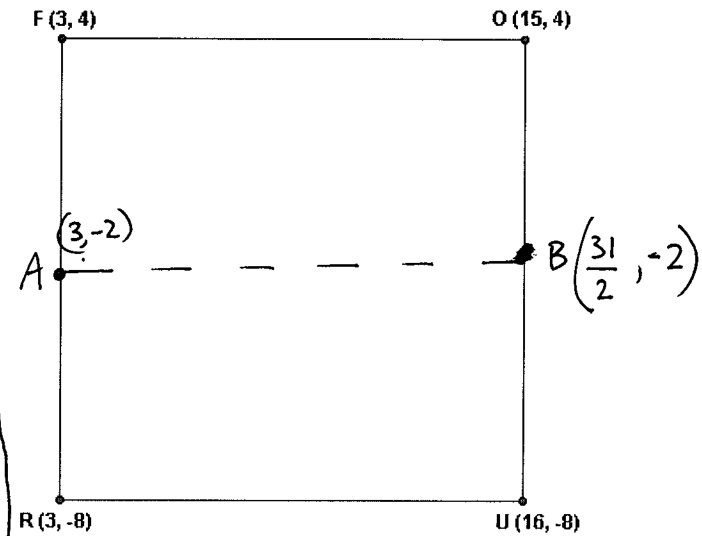
$-2 \cdot \frac{1}{2} = -1$ so $\overline{RO} \perp \overline{MH}$ and ∠1 + ∠2 are then rt. ∠s. Our rt. Δs now already have ≅ hypotenuses from steps above.

$RB = \sqrt{(6-7)^2+(6-4)^2}$
$= \sqrt{(-1)^2+(2)^2}$
$= \sqrt{5}$

$BO = \sqrt{(7-8)^2+(4-2)^2}$
$= \sqrt{(-1)^2+(2)^2}$
$= \sqrt{5}$

→ Δ RBH ≅ Δ OBM by HL ✓

3. Being as specific as possible, what type of figure is this? prove it.



slope $\overline{FO} = \frac{4-4}{15-3} = 0$

slope $\overline{RU} = \frac{-8--8}{3-16} = 0$

$\rightarrow \overline{FO} \parallel \overline{RU}$

slope $\overline{FR} = \frac{4--8}{3-3} = \frac{12}{0}$ = undefined = vertical line

slope $\overline{OU} = \frac{4--8}{15-16} = \frac{12}{-1} = -12$

$\rightarrow$ lines not $\parallel$

$FR = 12$ (don't need distance formula for vertical lines - too easy)

$OU = \sqrt{(15-16)^2 + (4--8)^2}$

$= \sqrt{(-1)^2 + (12)^2}$

$= \sqrt{145}$

$FR \neq OU$ so

$\rightarrow$ it's a trapezoid, (not an isos. trap.) ✓

$\left(\frac{15+16}{2}, \frac{4+-8}{2}\right) = \left(\frac{31}{2}, -2\right)$

Find the midpoints of $\overline{FR}$ and $\overline{OU}$ and label them A and B, respectively.

$\left(\frac{3+3}{2}, \frac{4+-8}{2}\right) = (3, -2)$

Find AB.

$= \sqrt{\left(3-\frac{31}{2}\right)^2 + (-2--2)^2} = \sqrt{\left(\frac{-25}{2}\right)^2} = \frac{\pm 25}{2}$

Show that AB = ½ of (RU + FO) and parallel to it.

RU = 13, FO = 12

$13 + 12 = 25$

$\frac{1}{2}(25) = \frac{25}{2}$ ✓

1. con't'd

$$m\overline{BD} = \frac{3--4}{-4-7} = \frac{-7}{11} \longrightarrow y-3 = \frac{-7}{11}(x--4)$$

$$y = \frac{-7}{11}x - \frac{28}{11} + 3$$

$$y = \frac{-7}{11}x + \frac{5}{11}$$

$$m\overline{AC} = \frac{-1-0}{-5-8} = \frac{1}{13} \longrightarrow y-0 = \frac{1}{13}(x-8)$$

$$y = \frac{1}{13}x - \frac{8}{13}$$

$$\frac{-7}{11}x + \frac{5}{11} = \frac{1}{13}x - \frac{8}{13}$$

$$-91x + 65 = 11x - 88$$

$$153 = 102x$$

$$\frac{152}{102} = x$$

$$\frac{9}{6} = x$$

$$\frac{3}{2} = x$$

$$y = \frac{-7}{11}\left(\frac{3}{2}\right) + \frac{5}{11}$$

$$22y = -21 + 10$$

$$y = \frac{-11}{22} = \frac{-1}{2}$$

so, $E = \left(\frac{3}{2}, \frac{-1}{2}\right)$

$$BE = \sqrt{\left(-4-\frac{3}{2}\right)^2 + \left(3--\frac{1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-11}{2}\right)^2 + \left(\frac{7}{2}\right)^2} = \sqrt{\frac{121}{4} + \frac{49}{4}} = \sqrt{\frac{170}{4}} = \frac{\sqrt{170}}{2}$$

so, $BE = \frac{1}{2}BD$ and E is m.p. of $\overline{BD}$

$$AE = \sqrt{\left(-5-\frac{3}{2}\right)^2 + \left(-1--\frac{1}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-13}{2}\right)^2 + \left(\frac{-1}{2}\right)^2}$$

$$= \sqrt{\frac{169}{4} + \frac{1}{4}} = \sqrt{\frac{170}{4}} = \frac{\sqrt{170}}{\sqrt{4}}$$

$$= \frac{\sqrt{170}}{2}$$

so, $AE = \frac{1}{2}AC$

and E is m.p. of $\overline{AC}$